COMMUNICATIONS TO THE EDITOR

Partial Condensation by the Methods of Simultaneous Heat and Mass Transfer

E. J. HOFFMAN

University of Wyoming, Laramie, Wyoming

Condensation or vaporization in systems of two or more components may be treated by the simultaneous relationships involving both heat and mass transfer. In addition to sensible heat changes accompanying mass transfer, there are also latent heat effects whenever change of phase is involved. The purpose here is to examine these relationships, particularly with respect to partial condensation involving a condensable and a noncondensable. At the same time, appropriate reductions are of concern in order that the set of equations be solvable.

EQUATIONS AND VARIABLES

The set of equations describing the system is composed of simple differential forms accompanied by algebraic balances and constraints, which may be expressed in several forms and arrangements. Involved are rate relations for heat and mass transfer, and energy and material balances, along with appropriate constraints.

Rate Equations

The rate relations with respect to the area A are

$$\frac{-d\,(Vy_i)}{dA} = (k_g)_i\,(\gamma_i - \gamma^*{}_i) \qquad (1)$$

$$\frac{-V\,dH_v}{dA} = h_g\,(T_v - T^*) \qquad (2)$$

The absolute activity at the interface would be defined from the equilibrium condition of the liquid phase as the interface (*1*):

$$\gamma_i^* = \text{func.}\,(T^*, P, x_1)$$

where x_i is the liquid composition at the interface and is considered the uniform bulk composition at a point A. The system pressure P is regarded as constant. For most practical purposes, $\gamma = Py_i$ and $\gamma_i^* = Py_i^* = vp_i\,x_i$.

The material balance for any component i is

$$-\,[Vy_i - V_1(y_i)_1] = Lx_i - L_1\,(x_i)_1 \qquad (3)$$

where l denotes inlet conditions. Most usually $L_1 = 0$. Furthermore, it is required that $\Sigma y_i = 1$ and $\Sigma x_i = 1$.

The overall energy balance may be written as

$$\pm\,W_m'\,[H' - H_1'] = (V_1 - L)\,H_v + LH_L - V_1\,(H_v)_1$$

$$= V_1\,[H_v - (H_v)_1] - L\,(H_v - H_L)$$

The sign in front of W_m' will henceforth have the following convention: the upper sign will indicate countercurrent flow. Thus, in the above expression, a positive sign will be used when the coolant flow W_m' is countercurrent to the vapor flow. If it is agreed that $(H_v - H_L) \sim \lambda_m$, the latent heat of the mixture at a point and the heat capacities are taken as constant averages,

$$\pm\,W_m'\,[H' - H_1'] = V_1\,\overline{C}_{p_v}\,[T_v - (T_v)_1] - L\lambda_m \qquad (4)$$

The rate equation for heat transfer to the coolant is

$$\pm\,W_m'\,\frac{dH}{dA} = U'\,(T_w - T')$$

where

$$U' = \frac{1}{\dfrac{1}{h_0} + R_{w0} + R_{F0}}$$

The subscript 0 will denote that the coefficients are based on the area A, including the wall and fouling resistances.

If heat transfer effects are to be considered for the condensate film, then a form of the Nusselt relations for heat transfer across a viscous film should be included.

Of the several possible forms for the Nusselt relations, the following are representative. They are arranged to be expressed directly in terms of linear condensate rate Γ by combining the condensing coefficient with the overall heat balance, as outlined by Hoffman (*2*).

For vertical tubes or surfaces:

$$\frac{\Gamma^{4/3}}{\dfrac{\overline{U'}\,\Delta L}{\Delta h}\,(T^* - T') - \Gamma} = \frac{0.924\left[\dfrac{k^3\rho^2 g}{\mu}\right]^{1/3}}{U'}$$

For horizontal tubes or surfaces:

$$\frac{\Gamma^{4/3}}{\dfrac{\overline{U'}\,N\Pi D}{\Delta h}\,(T^* - T') - \Gamma} = \frac{0.9532\left[\dfrac{k^3\rho^2 g}{\mu}\right]^{1/3} n_e^{1/3}}{U'}$$

where the effective number of vertical rows (each of n_i tubes) is

$$n_e = \left[\frac{(\Sigma n_i^{3/4})^{4/3}}{N} \right]^3$$

and is related to the average number of tubes per vertical row by

$$n_e \, n_a = N$$

where

$$n_a = \left[\frac{N}{\Sigma n_i^{3/4}} \right]^4$$

The above expressions involving Γ are of the form

$$\frac{\Gamma^{4/3}}{\Phi - \Gamma} = \psi \tag{5}$$

where Φ and ψ would be defined as indicated.

Whenever the Nusselt relations are to be applied, a simplification can be made. Consider Equation (5). Dividing both sides by ψ will give

$$\frac{(\Gamma')^{4/3}}{\Phi' - \Gamma'} = 1$$

where

$$\Gamma' = \frac{\Gamma}{\psi^3}, \quad \Phi' = \frac{\Phi}{\psi^3}$$

A plot of this equation indicates that over suitable intervals

$$\Gamma' \sim a\,(\Phi')^b \quad \text{or} \quad \Gamma = a\,\psi^{3(1-b)}\,\Phi^b$$

where a and b are constants for the interval.

Thus, for a particular geometry and fluid of molecular weight M_L,

$$\Gamma = -\frac{dV}{dA} M_L \frac{A}{\Delta L} = a\,\psi^{3(1-b)} \left[\frac{U'}{\Delta h} \frac{A}{\Delta L} (T^* - T') \right]^b$$

where ΔL is tube length. Hence, Equation (5) becomes

$$-\frac{dV}{dA}$$

$$= \left[a\,\psi^{3(1-b)} \left(\frac{\Delta L}{A} \right)^{1-b} \frac{1}{M_L} \left(\frac{U'}{\Delta h} \right)^b \right] (T^* - T)^b$$

$$= \alpha\,(T^* - T')^b \tag{5}$$

where α would be defined as indicated and $\Delta h\ M_L \sim \lambda_m$.

PARTIAL CONDENSATION

If partial condensation is to be meant as the transfer of one component only, then for the condensable $x_i = 1$. Dropping subscripts for the condensable, the set of equation reduces to

$$\frac{-dV}{dA} = k_g\ P(y - y^*) \tag{1}$$

$$-V\ C_{pv} \frac{dT_v}{dA} = h_g\,(T_v - T^*) \tag{2}$$

$$y^* = \frac{vp^*}{P} \ (\text{or } vp^* = \text{func. } (T^*))$$

$$\frac{1 - y}{1 - y} = \frac{V_1}{V} \tag{3}$$

$$\pm W_m'\ \overline{C}_p'\ (T' - T_1') = V_1\ \overline{C}_{pv}\ [T_v - (T_v)_1] - (V_1 - V)\ \lambda_m \tag{4}$$

$$\frac{-dV}{dA} = \alpha\,(T^* - T')^b \tag{5}$$

The Lewis Relation

The area of transfer can be eliminated as a variable between Equations (1) and (2) for partial condensation. Also eliminating V as a variable results in the form

$$\frac{1}{P\,\overline{C}_{pv}} \frac{h_g}{k_g} \cdot \frac{1}{(1 - y)(y - y^*)} = \frac{d\,T_v}{T_v - T^*} \tag{1-2}$$

The ratio of the Schmidt to Prandtl number (or sometimes the inverse) is called the Lewis number, *Le*. For gases, this relationship can be reduced to

$$\frac{k_g}{h_g} = \frac{1}{P\,C_p} = \frac{1}{PMc_P} \quad \text{or} \quad \frac{1}{P\,C_p} \frac{h_g}{k_g} = 1$$

Further Elimination of Variables

If area of transfer is eliminated between Equations (1) and (2), it should also be eliminated from Equation (5). Thus combining Equations (1) and (5) will yield the algebraic equation

$$k_g\ P\,(y - y^*) = \alpha\,(T^* - T')^b \tag{1-5}$$

By combining Equations (3) and (4), the variable V may be eliminated. The variable T' can be eliminated between Equations (3) and (4) and (1) and (5). The set of equations for partial condensation reduce to

$$\frac{dy}{(1 - y)(y - y^*)} = \frac{dT_v}{T_v - T^*} \tag{1-2}$$

$$y^* = \frac{vp^*}{P} = \frac{a_i\,e^{-b_i/T^*}}{P}$$

$$\pm W_m'\ \overline{C}_p \left\{ T^* - T_1 - \left[\frac{k_g\,P}{\alpha} (y - y^*) \right]^{1/b} \right\}$$

$$= V_1\ \overline{C}_{pv}\ [T_v - (T_v)_1] + V_1 \frac{(y - y^*{}_1)}{1 - y} \lambda_m \tag{3-4, 1-5}$$

We have reduced the number of equations by three and the number of variables by three. The variable y^* can of course be eliminated by substituting the equilibrium relationship.

The problem is reduced to solving one differential equation simultaneously to one algebraic equation. Solution of the algebraic equation for T^* and substitution into the differential equation would, in principle, reduce the problem to one of solving a single differential equation in y and T_v. The solution for this set of equations then permits the determination of the behavior of the other variables from substitution into the other simultaneous equations. Finally, area is determinable by integration of Equations (1), (2), or (5).

The solution can be conveniently recorded as a plot of y vs. T_v and y^* vs. T^* on a y-T plot, and also as T_v, T^*, and T' versus A.

Boundary Conditions

At the inlet conditions

$$y = y_1, \quad T_v = (T_v)_1, \quad T' = T_1'$$

From Equations (1-5)

$$k_g\ P\,(y_1 - y^*) = \alpha\,(T_1^* - T_1')^b$$

where

$$y_1^* = \frac{vp_1^*}{P} = \frac{a_i\, e^{-b_i/T_1^*}}{P}$$

The simultaneous solution will yield the inlet values y_1^* and T_1^*.

At the inlet, a condition of fogging may be indicated if

$$\frac{dy}{dT_v} > \frac{dy^*}{dT^*}$$

This is particularly so if the inlet vapor is saturated.

SOLUTION OF THE EQUATIONS

Several alternative approaches exist to solving the resultant system of a differential and an algebraic equation. Basically, the problem here is to effect a numerical solution of the differential equation in two variables; a third variable in the equation is a known function of the two variables.

It is satisfactory to evaluate the successive values of T^* by increments using a differential form of the energy balance which can in general be arranged as

$$dT^* = \frac{\left\{ \frac{k_gP}{\alpha} Y_c + \frac{V_1\, \lambda_m}{W_m'\, C_{p1}} \frac{(1 - y_1)}{(1 - y)^2} \right\} dy \pm \frac{V_1\, C_{pv}}{W_m'\, C_p'} dT_v}{1 + \frac{k_gP}{\alpha} Y_c\, y^* \frac{b_i}{(T^*)^2}} \tag{6}$$

where

$$y^* = \frac{vp^*}{P} = a_i\, e^{-b_i/T^*}$$

$$Y_c = \frac{1}{b} \frac{k_gP}{\alpha} (y - y^*)^{\frac{1}{b} - 1}$$

Using the differential form, increments of T^* can be determined from increments of y and T_v. The variable T_v can be regarded a preferable choice for the independent variable in the approximation process.

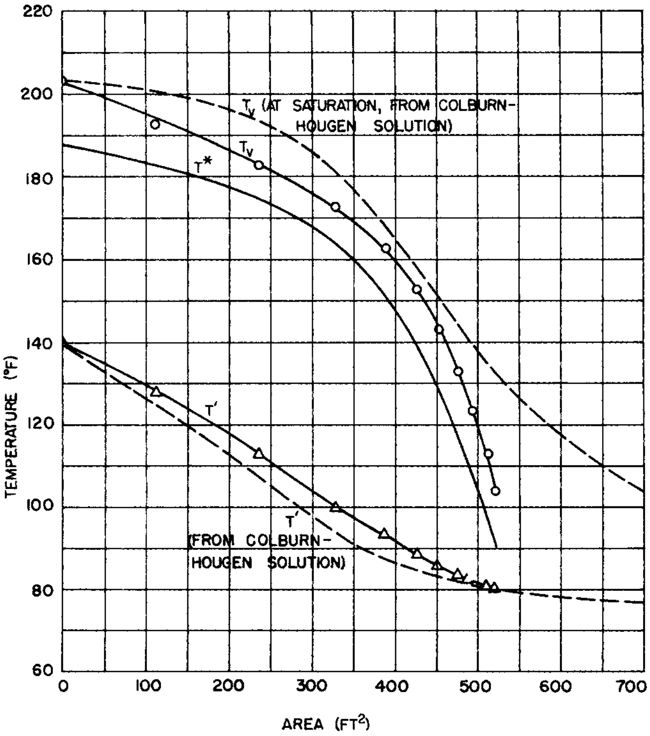

Fig. 1. Temperature versus area.

The solution will first appear as some combination between T_v, y, and T^*. For example, using T_v as the independent variable

$$y = y(T_v), \quad T^* = T^*(T_v)$$

The behavior of the variable y^* follows from the equilibrium relation. The behavior of the variable T' will then conveniently follow from rearrangement of Equation (1-5)

$$T' = T^* - \left[\frac{k_gP}{\alpha} (y - y^*) \right]^{1/b}$$

The variable V follows from the material balance

$$V = V_1 \frac{1 - y_1}{1 - y}$$

Colburn-Hougen Method

In the Colburn-Hougen method (*3*), Equations (1) and (2) would be substituted into the differential form of Equation (4), dropping an equation, so V is usually assumed at saturation. Various solutions to this set of equations are well documented in the literature and will not be dealt with here.

The now well-known problem, first used by Colburn and Hougen (*3*) for the partial condensation of water from a noncondensable gas, will be solved using the methods and conditions previously set forth. This particular problem has been well worked over in the literature, but it ordinarily used the Colburn-Hougen relation and assumes the vapor phase at a saturation condition. These conditions will not be applied here.

STATEMENT OF PROBLEM

A partial condenser is to cool and condense from a gaseous mixture consisting of 100 lb.-moles/hr. of inert gas (mole wt. 28) saturated with water vapor at 95°C. and one atm. The gaseous mixture is to be cooled to 40°C. Cooling water is available at 25°C., and is to leave at 60°C. Countercurrent flow is employed.

The exchanger is to be of a horizontal shell and tube construction with the cooling water on the tube side. The tubes are to be 0.75 in.-O.D. brass tubes of 16 B.W.G. (0.62 in. I.D.).

It is required to determine the area or tube length for an assumed number of tubes (and layout). For the coolant rate assumed by Colburn and Hougen, the number of tubes required would be equal to 83. On the other hand, Kern (*4*) lists a tube count of 92 in a 12-in. shell for 0.75-in. tubes on a 1-in. triangular pitch for a single-pass exchanger. This figure will be used.

Correlations by Devore (*5*) indicate a value of $n_a = 4.6$ for triangular pitch layouts involving a total of 92 tubes. Accordingly

$$n_e = \frac{92}{4.6} = 20.0$$

The values for the conversion between Γ and V are determined from

$$\Gamma = \frac{dV}{dA} (M_L) \cdot \frac{A}{\Delta L}$$

For the horizontal tubebank, $\frac{A}{\Delta L} = N\, \Pi\, D = 18.05$ ft.

Average mean physical properties will be assumed over the bundle, for air and water vapor mixtures, and for water.

Average or mean values will be assumed for h_g and k_g

TABLE 1. NUMERICAL SOLUTION FOR $\delta T_v = 0.1$°F.

T_v °F.	y	T^* °F.	y^*	T' °F.	V lb.-moles	A sq. ft.
203	0.835	187.5	0.617	140	606	0
193	0.801	182.7	0.554	128.0	502.8	112.4
183	0.734	174.6	0.461	113.2	376.1	235.7
173	0.634	163.7	0.357	101.1	273.3	329.8
163	0.525	151.5	0.265	93.5	210.5	388.6
153	0.424	139.1	0.194	88.9	173.5	426.8
143	0.336	127.1	0.142	85.8	150.5	454.2
133	0.261	116.0	0.104	83.7	135.3	476.1
123	0.199	105.9	0.078	82.2	124.8	495.3
113	0.148	97.2	0.061	81.0	117.3	513.8
104	0.109	90.5	0.050	80.1	112.2	531.5

$h_g = 34.5$ B.t.u./hr.-sq.ft.-°F.;

$$k_g = h_g \frac{1}{C_{p_v} P} = 4.1 \text{ moles/hr.-sq.ft.-atm.}$$

The tubeside water rate is specified as 154,000 lb./hr.

$$W_m' = \frac{154{,}000}{18} = 8550 \text{ lb./moles/hr.}$$

$$h_0 = 1200 \text{ B.t.u./hr.-sq.ft.-°F.}$$

Tube wall resistance will be neglected. The fouling coefficient is 700 B.t.u./hr. sq.ft.-°F. On this basis $U' =$ 440 B.t.u./hr.-sq.ft.-°F.

The value of ψ is determined to be 23.2:

$$b = 0.859, \quad a = 0.515, \quad \alpha = 0.0236$$

Vapor pressure constants are

$$a_i = 45.5\ (e^{10}), \quad b_i = 9260$$

The vapor inlet conditions are

$$(T_V)_1 = 95\text{°C.} = 203.0\text{°F.}$$

$$y_1 = \frac{vp}{P} = \frac{633.90/760}{(1)} = 0.835$$

$$V_1 = \frac{100}{1 - y_1} = 606 \text{ lb. moles/hr.}$$

The vapor outlet temperature is specified at $(T_V)_2 =$ 40°C. = 104°F. The cooling water is to flow countercurrent to the vapor. The inlet temperature for the water is specified at $T_2' = 25$°C. = 77°F.

At the vapor inlet, T^* and y^* are determinable from the simultaneous solution of Equation (1-5) and the vapor pressure relation

$$T_1^* = 187.5\text{°F.}, \quad y_1^* = 0.617$$

NUMERICAL INTEGRATION

Equations (1-2) and (6) are integrated numerically for increments of T_v of 0.1°F. Reducing the increment to 0.06°F. showed negligible change. The behavior of T' and V follows, and that of A from an integration. The results are presented in Table 1 and Figure 1.

The condensables are nearly all condensed. The inlet water temperature was specified as 77°F. instead of 80.1°F. as calculated. This specification can be met by lowering T' from 140°F.

The methods utilized may also be applied to fouling (6) and crystallization (7).

ACKNOWLEDGMENT

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NOTATION

a, b = constants for Nusselt fit
a_i, b_i = constants for vapor pressure fit
A = condenser surface area
c_p = heat capacity (mass basis)
C_p = heat capacity (mole basis)
D = tube diameter
h_g = coefficient of heat transfer
h_c = condensing coefficient
h_0 = tubeside convection coefficient
H_v = molar enthalpy of vapor
H_L = molar enthalpy of condensate
H' = molar enthalpy of coolant
Δh = enthalpy difference between vapor and condensate (mass basis)
k_g = coefficient of mass transfer (in absolute activity units)
k = thermal conductivity
L = molar condensate rate
M = molecular weight
n_i = number of tubes in a vertical row "i"
n_e = effective number of vertical tuberous
n_a = average number of tubes per vertical row
N = total number of tubes
R_{w0} = tubewall resistance
R_{F0} = fouling resistance
T (or T_v) = vapor phase temperature
T^* = interface temperature
T' = coolant temperature
T_w = outer tubewall temperature
U' = heat transfer coefficient between T_w and T'
vp or vp^* = vapor pressure
V = molar vapor rate
W_m' = coolant rate (mole basis)
x_i = mole fraction in condensate
y_i = mole fraction in vapor phase
y_i^* = mole fraction at interface

Greek Letters

α = combined constant for condensate film
γ_i = absolute activity in gas phase
γ_i^* = absolute activity at interface
Γ' = Γ/ψ^3
λ_m = latent heat (mole basis)
μ = viscosity of condensate
ρ = density of condensate
Φ = group in Nusselt transformation
Φ' = Φ/ψ^3
ψ = group in Nusselt transformation

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